

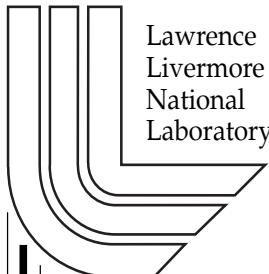
Axions from Wall Decay

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Axions from wall decay

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We discuss the decay of axion walls bounded by strings and present numerical simulations of the decay process. In these simulations, the decay happens immediately, in a time scale of order the light travel time, and the average energy of the radiated axions is $\langle \omega_a \rangle \simeq 7m_a$ for $v_a/m_a \simeq 500$. $\langle \omega_a \rangle$ is found to increase approximately linearly with $\ln(v_a/m_a)$. Extrapolation of this behaviour yields $\langle \omega_a \rangle \simeq 60m_a$ in axion models of interest.

1. Introduction

The axion is the quasi-Nambu-Goldstone boson associated with the spontaneous breaking of the $U_{PQ}(1)$ symmetry which Peccei and Quinn postulated [1]. Its zero temperature mass is given by:

$$m_a \simeq 6 \cdot 10^{-6} \text{ eV} \cdot N \cdot \left(\frac{10^{12} \text{ GeV}}{v_a} \right) \quad (1)$$

where v_a is the magnitude of the vacuum expectation value that breaks $U_{PQ}(1)$ and N is a positive integer which describes the color anomaly of $U_{PQ}(1)$. The axion owes its mass to non-perturbative QCD effects. At temperatures high compared to QCD scale, these effects are suppressed and the axion mass is negligible. There is a critical time t_1 , such that $m_a(t_1)t_1 = 1$, when the axion mass effectively turns on. The corresponding temperature $T_1 \simeq 1 \text{ GeV}$.

In case that inflation occurs with reheat temperature higher than v_a , axion strings form and radiate axions till t_1 . At t_1 each string becomes the boundary of N domain walls. If $N = 1$, the network of walls bounded by strings is unstable and decays away. There are in this case three contributions to the axion cosmological energy density: axions that were radiated by axion strings before t_1 , axions from vacuum realignment, and axions that were produced in the decay of walls bounded by strings after t_1 . Here, we discuss the third of these contributions.

That axions walls bounded by string decay predominantly into barely relativistic axions was

suggested in Ref. [2]. D. Lyth [3] also discussed the wall decay contribution and emphasized the uncertainties affecting it. Nagasawa and Kawasaki [4] performed computer simulations of the decay of walls bounded by string and obtained $\langle \omega_a \rangle / m_a \simeq 3$ for the average energy of the radiated axions. Our simulations, presented in section 2, are similar to those of Ref. [4] but done on larger lattices and for a wider variety of initial conditions. We obtain $\langle \omega_a \rangle / m_a \simeq 7$ where $v_a/m_a \simeq 500$. We attribute the difference between our result and that of Ref. [4] to the fact that we give angular momentum to the collapsing walls.

The lattice sizes which are amenable to present day computers are at any rate small compared to what one would ideally wish. The lattice constant must be smaller than $\delta \equiv (\sqrt{\lambda} v_a)^{-1}$ to resolve the string core. On the other hand the lattice size must be larger than m_a^{-1} to contain at least one wall. Hence the lattice size in units of the lattice constant must be of order $\frac{10}{m_a \delta} \times \frac{10}{m_a \delta}$ or larger if the simulations are done in 2 dimensions. Present day computers allow lattice sizes of order 4000×4000 , i.e. $\frac{\sqrt{\lambda} v_a}{m_a} \sim 100$. In axion models of interest $\frac{\sqrt{\lambda} v_a}{m_a} \sim \frac{10^{12} \text{ GeV}}{10^{-5} \text{ eV}} = 10^{26}$. To remedy this shortcoming, we study the dependence of $\langle \omega_a \rangle / m_a$ upon the $\frac{\sqrt{\lambda} v_a}{m_a}$ and find that it increases approximately as the logarithm of that quantity; see section 2.

The following toy model incorporates the qualitative features of interest:

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}\partial_\mu\phi^\dagger\partial^\mu\phi - \frac{\lambda}{4}(\phi^\dagger\phi - v_a^2)^2 + \eta v_a\phi_1 \\ &= \frac{1}{2}\partial_\mu\phi_1\partial^\mu\phi_1 + \frac{1}{2}\partial_\mu\phi_2\partial^\mu\phi_2 \\ &\quad - \frac{\lambda}{4}(\phi_1^2 + \phi_2^2 - v_a^2)^2 + \eta v_a\phi_1\end{aligned}\quad (2)$$

where $\phi = \phi_1 + i\phi_2$ is a complex scalar field. When $\eta = 0$, this model has a $U(1)$ global symmetry under which $\phi(x) \rightarrow e^{i\alpha}\phi(x)$. It is analogous to the $U_{PQ}(1)$ symmetry of Peccei and Quinn. It is spontaneously broken by the vacuum expectation value $\langle\phi\rangle = v_a e^{i\alpha}$. The associated Nambu-Goldstone boson is the axion. The last term in Eq. (2) represents the non-perturbative QCD effects that give the axion its mass m_a . We have $m_a = \sqrt{\eta}$ for $v_a \gg m_a$.

When $\eta = 0$, the model has global string solutions. A straight global string along the \hat{z} -axis is the static configuration:

$$\phi(\vec{x}) = v_a f(\rho) e^{i\theta} \quad (3)$$

where (z, ρ, θ) are cylindrical coordinates and $f(\rho)$ is a function which goes from zero at $\rho = 0$ to one at $\rho = \infty$ over a distance scale of order $\delta \equiv (\sqrt{\lambda}v_a)^{-1}$. The energy per unit length of the global string is

$$\mu \simeq 2\pi \int_\delta^L \rho d\rho \frac{1}{2} |\vec{\nabla}\phi|^2 = \pi v_a^2 \ln(\sqrt{\lambda}v_a L) \quad (4)$$

where L is a long-distance cutoff. For a network of strings with random directions, as would be present in the early universe, L is of order the distance between strings.

When $\eta \neq 0$, the model has domain walls. If $v_a \gg m_a$, the low energy effective Lagrangian is :

$$\mathcal{L}_a = \frac{1}{2}\partial_\mu a\partial^\mu a + m_a^2 v_a^2 \left[\cos \frac{a}{v_a} - 1 \right]. \quad (5)$$

Its equation of motion has the well-known Sine-Gordon soliton solution :

$$\frac{a(y)}{v_a} = 2\pi n + 4\tan^{-1} \exp(m_a y) \quad (6)$$

where y is the coordinate perpendicular to the wall and n is any integer. Eq. (6) describes a domain wall which interpolates between neighboring vacua in the low energy effective theory (5).

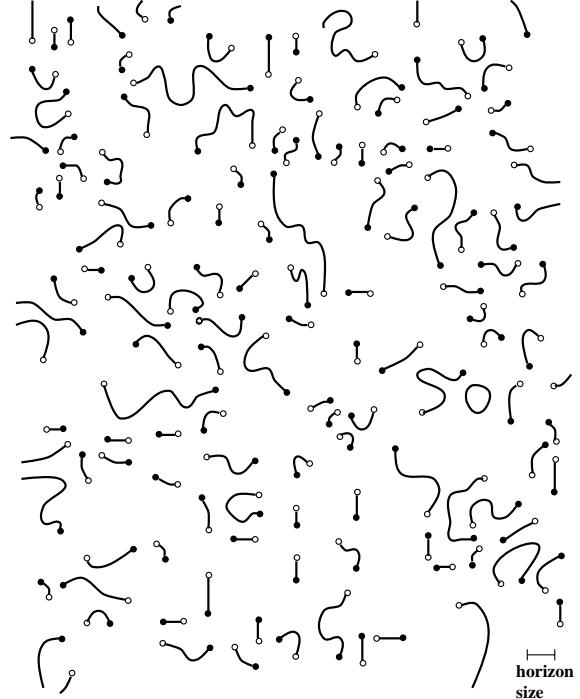


Figure 1. A small cross-section of the universe near the QCD phase transition, in $N = 1$ axion models. Lines are domain walls, open (filled) circles are upward (downward) going strings.

The thickness of the wall is of order m_a^{-1} . The wall energy per unit surface is $\sigma = 8m_a v_a^2$ in the toy model. In reality the structure of axion domain walls is more complicated than in the toy model, mainly because not only the axion field but also the neutral pion field is spatially varying inside the wall [5]. When this is taken into account, the (zero temperature) wall energy/surface is found to be:

$$\sigma \simeq 4.2 f_\pi m_\pi f_a \simeq 9 m_a f_a^2 \quad (7)$$

with $f_a \equiv v_a/N$.

In the early universe, the strings are stuck in the plasma and are stretched by the Hubble expansion. However with time the plasma becomes more dilute and below temperature [6]

$$T_* \sim 2 \cdot 10^7 \text{ GeV} \left(\frac{v_a}{10^{12} \text{ GeV}} \right)^2 \quad (8)$$

the strings move freely and decay efficiently into axions. Because this decay mechanism is very efficient, there is approximately one string per horizon

zon from temperature T_* to temperature $T_1 \simeq 1$ GeV when the axion acquires mass.

A cross-section of a finite but statistically significant volume of the universe near time t_1 was simulated with the assumption that the axion field is randomly oriented from one horizon to the next just before the axion mass turns on at time t_1 [7,8]. Fig. 1 shows the result of this simulation. The average density of walls at time t_1 predicted is approximately 2/3 per horizon volume at time t_1 .

2. Computer Simulations

We have carried out an extensive program of 2D numerical simulations of domain walls bounded by strings. The Lagrangian (2) in finite difference form is

$$\begin{aligned} L = & \sum_{\vec{n}} \left\{ \frac{1}{2} \left[\left(\dot{\phi}_1(\vec{n}, t) \right)^2 + \left(\dot{\phi}_2(\vec{n}, t) \right)^2 \right] \right. \\ & - \sum_{j=1}^2 \frac{1}{2} \left[\left(\phi_1(\vec{n} + \hat{j}, t) - \phi_1(\vec{n}, t) \right)^2 \right. \\ & + \left. \left(\phi_2(\vec{n} + \hat{j}, t) - \phi_2(\vec{n}, t) \right)^2 \right] \\ & - \frac{1}{4} \lambda \left[(\phi_1(\vec{n}, t))^2 + (\phi_2(\vec{n}, t))^2 - 1 \right]^2 \\ & \left. + \eta (\phi_1(\vec{n}, t) - 1) \right\} \end{aligned} \quad (9)$$

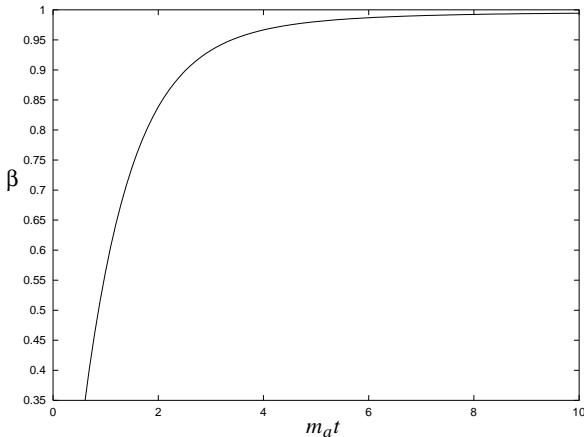


Figure 2. Speed of the string core as a function of time for the case $1/m_a = 400$, $\sqrt{\lambda}/m_a = 10$, and $v = 0$.

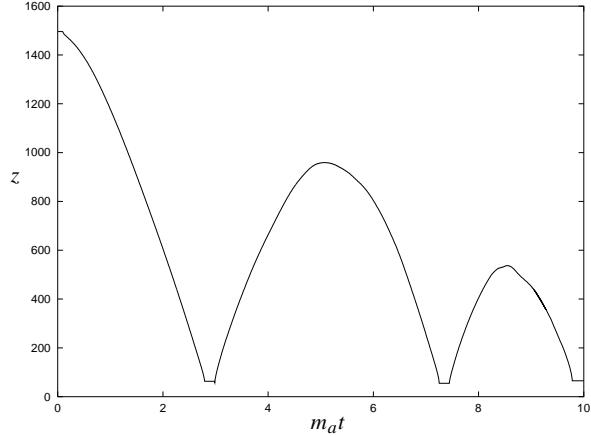


Figure 3. Position of the string or anti-string core as a function of time for $1/m_a = 1000$, $\lambda = 0.0002$, $D = 2896$, and $v = 0$. The string and anti-string cores have opposite z . They go through each other and oscillate with decreasing amplitude.

where \vec{n} labels the sites. In these units $v_a = 1$, the wall thickness is $1/m_a = 1/\sqrt{\eta}$ and the core size is $\delta = 1/\sqrt{\lambda}$. The lattice constant is the unit of length. In the continuum limit, the dynamics depends upon a single critical parameter, $m_a \delta = m_a/\sqrt{\lambda}$. Large two-dimensional grids ($\sim 4000 \times 4000$) were initialized with a straight domain wall initially at rest or with angular momentum. The initial domain wall was obtained by overrelaxation starting with the Sine-Gordon ansatz $\phi_1 + i\phi_2 = \exp(i \tan^{-1} \exp(m_a y))$ inside a strip of length D between the string and anti-string and the true vacuum ($\phi_1 = 1$, $\phi_2 = 0$) outside. The string and antistring core were approximated by $\phi_1 + i\phi_2 = -\tanh(.583 r/\delta) \exp(\mp i\theta)$ where (r, θ) are polar coordinates about the core center, and held fixed during relaxation. Stable domain walls were obtained for $1/(\delta m_a) \gtrsim 3$. A first-order in time and second order in space algorithm was used for evolution with a time step $dt = 0.2$. The boundary conditions were periodic and the total energy was conserved to better than 1%. If the angular momentum was nonzero, then the time derivative $\dot{\phi}$ was obtained by a finite difference over a small time step.

The evolution of the domain wall was studied for various values of $\sqrt{\lambda}/m_a$, the initial wall length D and the initial velocity v of the strings in

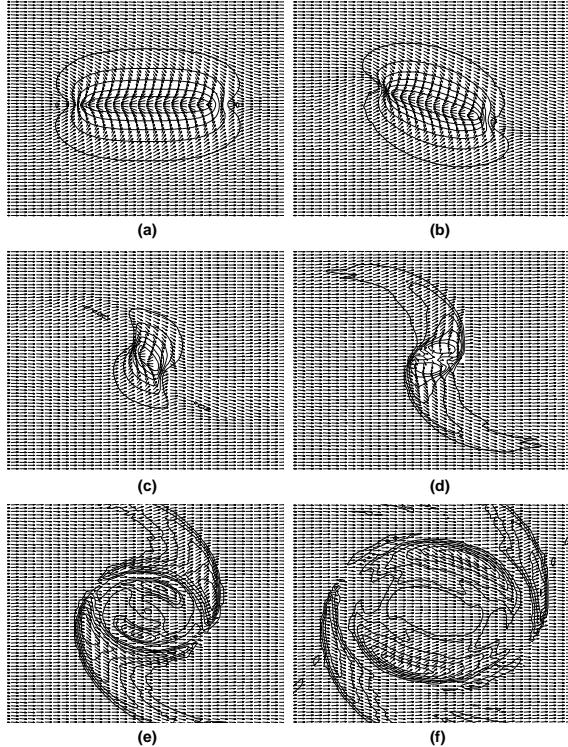


Figure 4. Decay of a wall at successive time intervals $\Delta t = 1.2/m_a$ for the case $m_a^{-1} = 100$, $\lambda = 0.01$, $D = 524$, and $v = 0.6$.

the direction transverse to the wall. Fig. 2 shows the longitudinal velocity of the core as a function of time for the case $m_a^{-1} = 400$, $\sqrt{\lambda}/m_a = 10$ and $v = 0$. An important feature is the Lorentz contraction of the core. For reduced core sizes $\delta/\gamma_s \lesssim 5$, where γ_s is the Lorentz factor associated with the speed of the string core, there is ‘scraping’ of the core on the lattice accompanied by emission of spurious high frequency radiation. This artificial friction eventually balances the wall tension and leads to a terminal velocity. In our simulations we always ensured being in the continuum limit.

For a domain wall without rotation ($v = 0$), the string cores strike head-on and go through each other. Several oscillations of decreasing magnitude generally occur before annihilation. For $\gamma_s \simeq 1$ the string and anti-string coalesce and annihilate one another. For $\gamma_s \gtrsim 2$, the strings go through each other and regenerate another

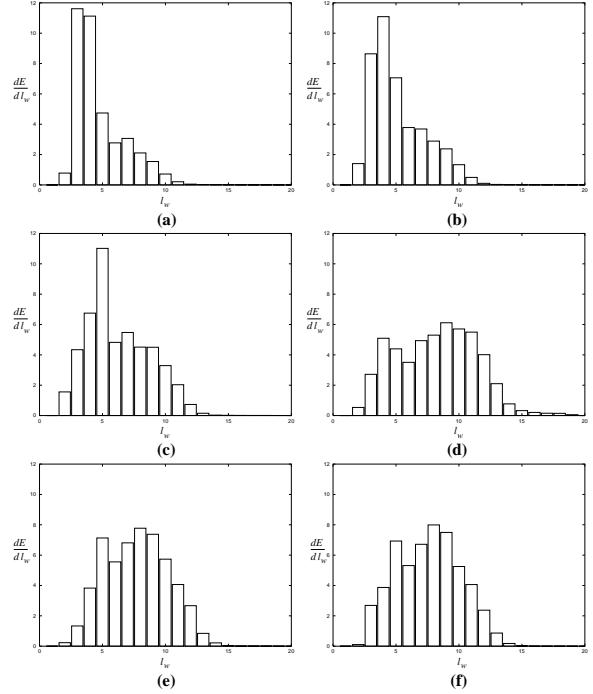


Figure 5. Energy spectrum at successive time intervals $\Delta t = 1/m_a$, with $w_{\max} = \sqrt{8 + m_a^2}$ and $l_w = 18 \ln(w/m_a)/\ln(w_{\max}/m_a) + 2$, for the case $m_a^{-1} = 500$, $\lambda = 0.0032$, $D = 2096$ and $v = 0.25$.

wall of reduced length. The relative oscillation amplitude decreases with decreasing collision velocity. Fig. 3 shows the core position as a function of time for the case $m_a^{-1} = 1000$, $\sqrt{\lambda}/m_a = 14.3$, $D = 2896$.

We also investigated the more generic case of a domain wall with angular momentum. The strings are similarly accelerated by the wall but string and anti-string cores miss each other. The field is displayed in Fig. 4 at six time steps for the case $m_a^{-1} = 100$, $\lambda = 0.01$, $D = 524$ and $v = 0.6$. No oscillation is observed. All energy is converted into axion radiation during a single collapse.

We performed spectrum analysis of the energy stored in the ϕ field as a function of time using standard Fourier techniques. The two-dimensional Fourier transform is defined by

$$\tilde{f}(\vec{p}) = \frac{1}{\sqrt{V}} \sum_{\vec{n}} \exp \left[2i\pi \left(\frac{p_x n_x}{L_x} + \frac{p_y n_y}{L_y} \right) \right] f(\vec{n}) \quad (10)$$

where $V \equiv L_x L_y$. The dispersion law is:

$$\omega_p = \sqrt{2 \left(2 - \cos \frac{2\pi p_x}{L_x} - \cos \frac{2\pi p_y}{L_y} \right) + m_a^2}. \quad (11)$$

Fig. 5 shows the power spectrum $dE/d \ln \omega$ of the ϕ field at various times during the decay of a rotating domain wall for the case $m_a^{-1} = 500$, $\lambda = 0.0032$, $D = 2096$, and $v = 0.25$. Initially, the spectrum is dominated by small wavevectors, $k \sim m_a$. Such a spectrum is characteristic of the domain wall. As the wall accelerates the string, the spectrum hardens until it becomes roughly $1/k$ with a long wavelength cutoff of order the wall thickness $1/m_a$ and a short wavelength cutoff of order the reduced core size δ/γ_s . Such a spectrum is characteristic of the moving string.

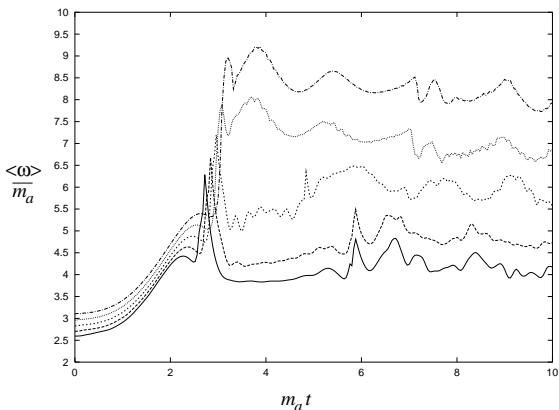


Figure 6. $\langle \omega \rangle$ as a function of time for $1/m_a = 500$, $D = 2096$, $v = 0.25$ and $\lambda = 0.0004$ (solid), 0.0008 (long dash), 0.0016 (short dash), 0.0032 (dot) and 0.0064 (dot dash). After the wall has decayed, $\langle \omega \rangle$ is the average energy of radiated axions.

Fig. 6 shows the time evolution of $\langle \omega \rangle / m_a$ for $m_a^{-1} = 500$, $v = 0.25$, $D = 2096$ and various values of λ . By definition,

$$\langle \omega \rangle = \sum_{\vec{p}} E_{\vec{p}} / \sum_{\vec{p}} \frac{E_{\vec{p}}}{\omega_{\vec{p}}} \quad (12)$$

where $E_{\vec{p}}$ is the gradient and kinetic energy stored in mode \vec{p} of the field ϕ . After the domain wall has decayed into axions, $\langle \omega \rangle = \langle \omega_a \rangle$. Fig. 7 shows the time evolution of $\langle \omega \rangle / m_a$ for $m_a^{-1} = 500$, $\lambda = 0.0016$, $D = 2096$ and various values of v .

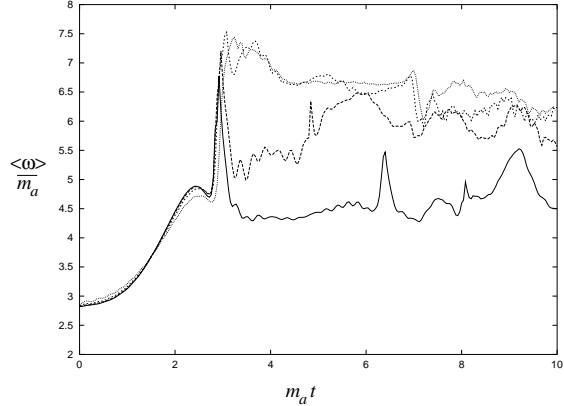


Figure 7. Same as in Fig. 6 except $\lambda = 0.0016$, and $v = 0.15$ (solid), 0.25 (long dash), 0.4 (short dash) and 0.6 (dot).

When the angular momentum is low and the core size is big, the strings have one or more oscillations. In that case, $\langle \omega_a \rangle / m_a \simeq 4$, which is consistent with Ref. [4]. We believe this regime to be less relevant to wall decay in the early universe because it seems unlikely that the angular momentum of a wall at the QCD epoch could be small enough for the strings to oscillate.

In the more generic case when no oscillations occur, $\langle \omega_a \rangle / m_a \simeq 7$. Moreover, we find that $\langle \omega_a \rangle / m_a$ depends upon the critical parameter $\sqrt{\lambda}/m_a$, increasing approximately as the logarithm of that quantity; see Fig. 6. This is consistent with the time evolution of the energy spectrum, described in Fig. 4. For a domain wall, $\langle \omega \rangle \sim m_a$ whereas for a moving string $\langle \omega \rangle \sim m_a \ln(\sqrt{\lambda} v_a \gamma_s / m_a)$. Since we find the decay to proceed in two steps: 1) the wall energy is converted into string kinetic energy, and 2) the strings annihilate without qualitative change in the spectrum, the average energy of radiated axions is $\langle \omega_a \rangle \sim m_a \ln(\sqrt{\lambda} v_a / m_a)$. Assuming this is a correct description of the decay process for $\sqrt{\lambda} v_a / m_a \sim 10^{26}$, then $\langle \omega_a \rangle / m_a \sim 60$ in axion models of interest.

3. Conclusions.

We have presented results of our computer simulations of the motion and decay of walls bounded by strings which appear during the QCD phase

transition in axion models with $N = 1$ assuming the axion field is not homogenized by inflation. In the simulations, the walls decay immediately, i.e. in a time scale of order the light travel time. The simulations provide an estimate of the average energy of the axions emitted in the decay of the walls: $\langle \omega_a \rangle \simeq 7m_a$ when $\sqrt{\lambda}v_a/m_a \simeq 20$.

Because of restrictions on the available lattice sizes, the simulations are for values of v_a/m_a of order 100, whereas in axion models of interest v_a/m_a is of order 10^{26} . To address this problem, we have investigated the dependence of $\langle \omega_a \rangle/m_a$ upon $\sqrt{\lambda}v_a/m_a$ and found that it increases approximately as the logarithm of this quantity. This is because the decay process occurs in two steps: 1) wall energy converts into moving string energy because the wall accelerates the string, and the energy spectrum hardens accordingly; 2) the strings annihilate into axions without qualitative change in the energy spectrum. If this behaviour persists all the way to $\sqrt{\lambda}v_a/m_a \sim 10^{26}$, then $\langle \omega_a \rangle/m_a \sim 60$ for axion models of interest.

This has interesting consequences for the axion cosmology and the cavity detector experiments of dark matter axion as discussed in Ref. [8].

Acknowledgments

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